UNIVERSITY OF ICELAND

Black Hole Informatics

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Why black holes?

Black holes exist in Nature

- end station of core collapse of sufficiently massive stars
- active galactic nuclei contain supermassive black holes
- important for gravitational wave astronomy
- ….

Black holes are interesting

- rich catalog of classical black hole solutions in GR and modified gravity theories
- extreme spacetime curvature indicates breakdown of classical theory
- quantum effects challenge effective field theory paradigm
- black hole thermodynamics hints at connection to quantum information theory
- string theory black holes connect number theory, geometry and physics
- ….

Summary

The information paradox highlights the incompatibility between general relativity (locality + equivalence principle) and quantum physics (unitarity).

Gauge theory - gravity correspondence implies unitary black hole evolution.

Black hole complementarity provides a "phenomenological" description, which preserves unitarity and the equivalence principle, but requires giving up locality.

Semiclassical gravity based on local effective field theory is nevertheless surprisingly effective

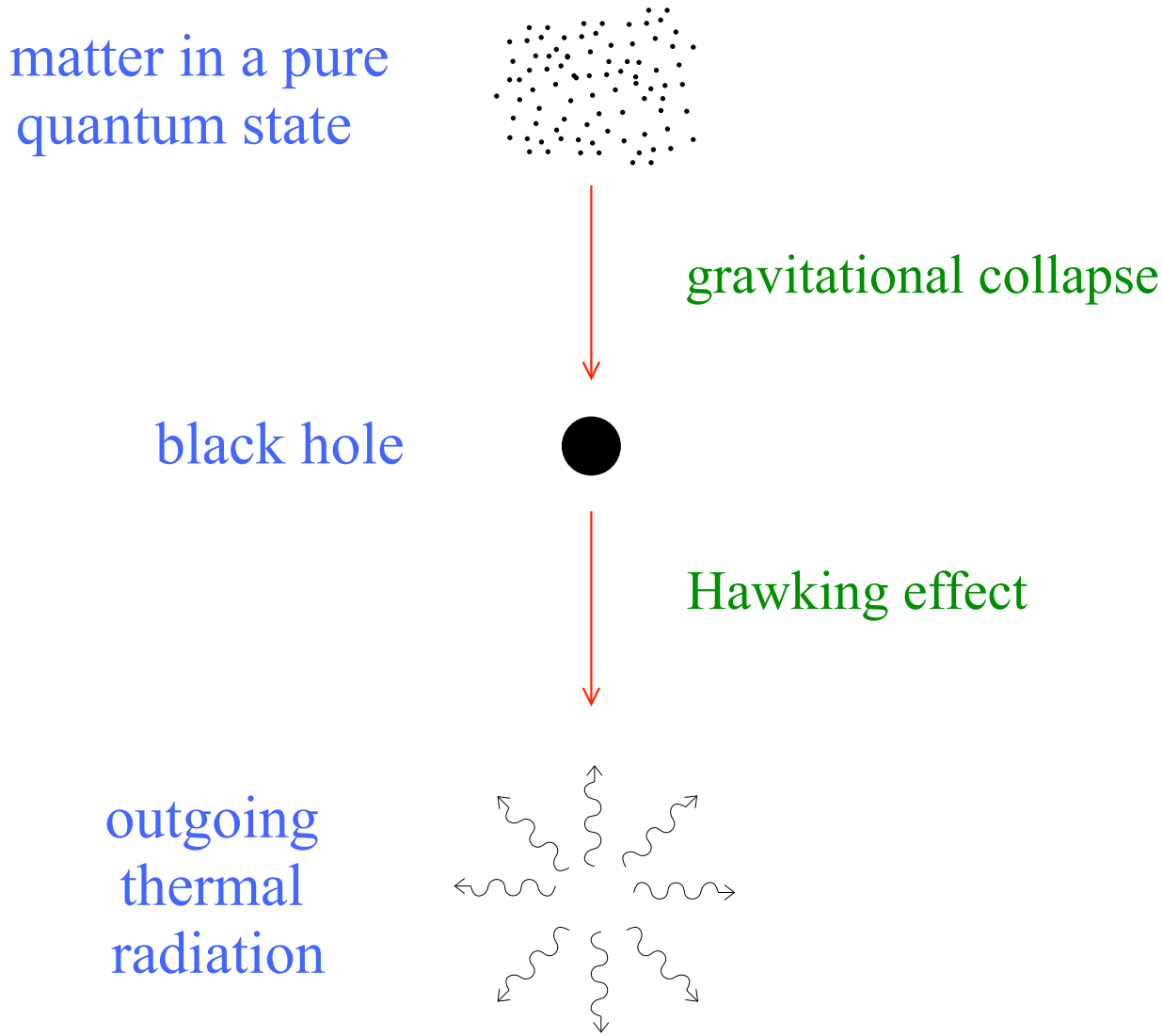
- Page curve reproduced via quantum extremal surface prescription
- rate growth of quantum complexity captured by (semi)classical BH geometry

So far, no satisfactory holographic model for the black hole interior

Requirements include:

- unitary time evolution of holographic d.o.f.'s
- an approximate description of observers in the black hole interior can be given in terms of an effective field theory, defined on a limited set of time slices, such that no drama is seen until near the singularity

Black hole information paradox Hawking 1976



Oxford Dictionary of English

informatics | ˌɪnfəˈmatɪks |

plural noun *[treated as singular] Computing*

the science of processing data for storage and retrieval;

Although there has been a lot of work in the last fifteen years [...],
I think it would be fair to say that we do not yet have a fully satisfactory
and consistent quantum theory of gravity.

Particle Creation by Black Holes
Stephen W. Hawking
April, 1975

Although much work remains to be done there seem to be no insuperable obstacles to deriving all of known physics from the $E_8 \times E_8$ heterotic string.

HETEROTIC STRING THEORY (I): The free heterotic string
David J. Gross, Jeffrey A. Harvey, Emil Martinec and Ryan Rohm
February, 1985

...the euclidean formulation of [quantum] gravity is not a subject with firm foundations and clear rules of procedure; indeed, it is more like a trackless swamp. I think I have threaded my way through it safely, but it is always possible that unknown to myself I am up to my neck in quicksand and sinking fast.

WHY THERE IS NOTHING RATHER THAN SOMETHING:
A theory of the cosmological constant
Sidney Coleman
May 1988

Schwarzschild black hole

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

There is a curvature singularity at r = 0 but the geometry is non-singular at the *event horizon* $r = 2M$

$$R_{\mu\nu\lambda\sigma}\, R^{\mu\nu\lambda\sigma} = \frac{48M^2}{r^6} \longrightarrow \begin{cases} \infty & \text{as} \quad r \to 0 \\ \frac{3}{4M^4} & \text{as} \quad r \to 2M \end{cases}$$

Spacetime is almost flat at the horizon of a large black hole!

$$R_{\mu\nu\lambda\sigma}\, R^{\mu\nu\lambda\sigma}\Big|_{r=2M} \longrightarrow 0 \quad \text{as} \quad M \to \infty$$

Natural units

- Constants of nature:

  $c = 2.998 \times 10^{8}$ m/s

  $\hbar = 1.055 \times 10^{-34}$ Js

  $G = 6.673 \times 10^{-11}$ Nm^2/kg^2

- Planck scale:

  $\ell_{\mathsf{Pl}} = \sqrt{\frac{G\hbar}{c^3}} = 1.6 \times 10^{-35}$ m

  $t_{\mathsf{Pl}} = \sqrt{\frac{G\hbar}{c^5}} = 5.4 \times 10^{-44}$ s

  $m_{\mathsf{Pl}} = \sqrt{\frac{\hbar c}{G}} = 2.2 \times 10^{-8}$ kg

  $E_{\mathsf{Pl}} = \sqrt{\frac{\hbar c^5}{G}} = 2.0 \times 10^{9}$ J $= 1.2 \times 10^{19}$GeV

Natural units: $c = \hbar = G = 1$

Measure velocity with respect to c,

Measure length with respect to ℓ_{Pl}, $etc.$

Global Schwarzschild spacetime

Kruskal-Szekeres coordinates

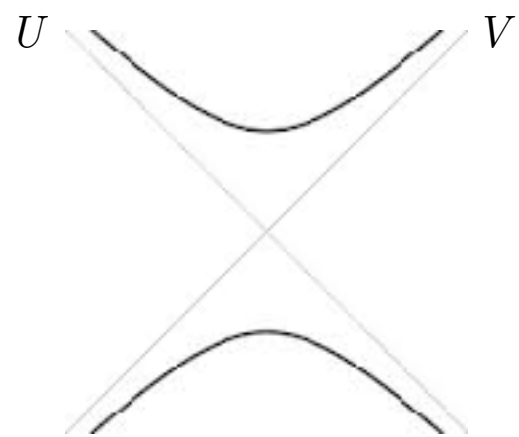
$$V = \left(\frac{r}{2M} - 1\right) \exp\left[\frac{r+t}{4M}\right], \qquad U = -\left(\frac{r}{2M} - 1\right) \exp\left[\frac{r-t}{4M}\right]$$

Metric is non-singular

$$ds^2 = -\frac{32M^3}{r} e^{-\frac{r}{2M}}\, dU\, dV + r^2 d\Omega^2$$

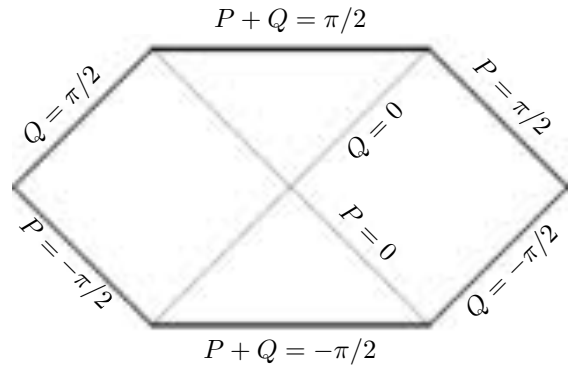
Event horizon is at $UV = 0$

Curvature singularity is at $UV = 1$



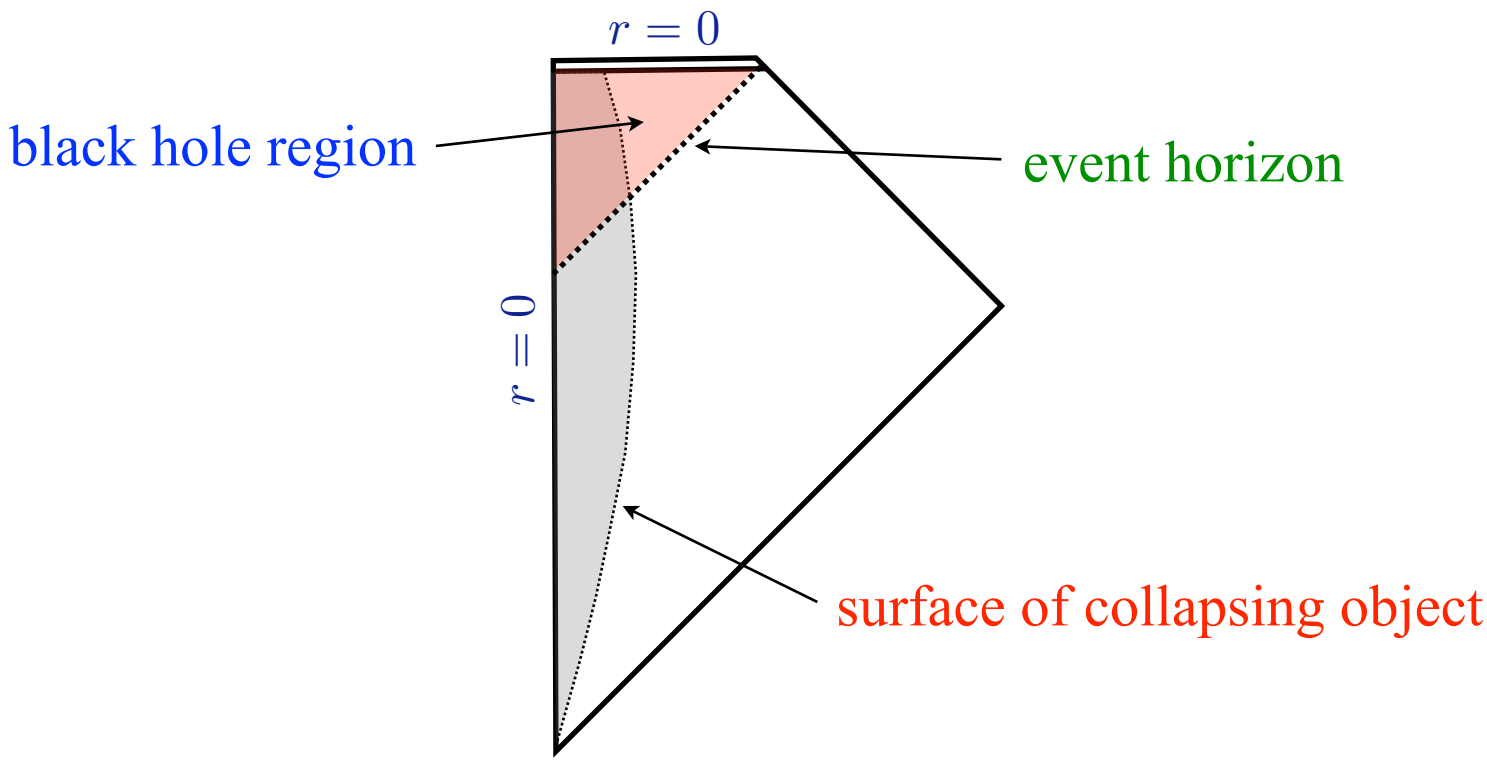
Penrose diagram

$$V = \tan P, \qquad U = \tan Q$$

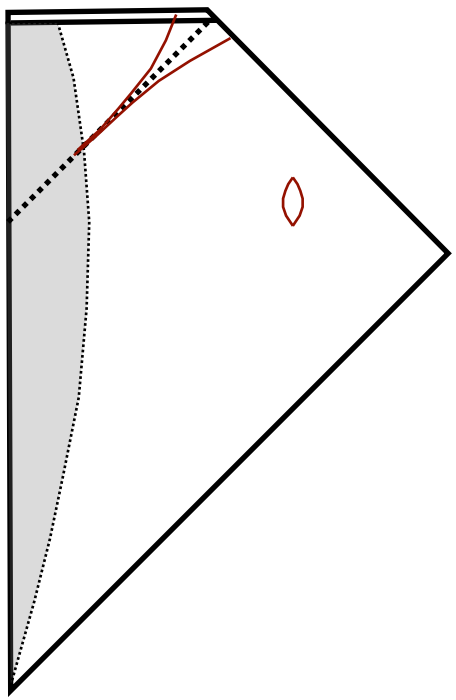


Black hole formed by gravitational collapse

- classical general relativity -



Hawking effect



Heuristic argument

Quantum mechanics: $\Delta E \Delta t \sim \hbar$

- virtual pair created in flat spacetime
- particle energies +*E* and -*E*
- particle with -*E* cannot propagate
- pair annihilates within time $\sim \hbar / E$

- virtual pair created across event horizon
- particle with -E propagates inside black hole
- particle with +E escapes to $r \to \infty$
- black hole loses mass

Semiclassical calculation Hawking 1974

Quantum field theory in curved spacetime

- quantise matter field in classical BH background
- mismatch between in- and out- vacuua
- outgoing particles at late times

Black hole evolution

According to Hawking's semi-classical calculation, black hole radiation is thermal

Hawking temperature: $$T_{\rm H} = 6 \times 10^{-8} \left(\frac{M_\odot}{M} \right) {\rm K}$$

Black hole lifetime: $$\tau \approx 10^{71} \left(\frac{M}{M_\odot} \right)^3 {\rm s}$$

$$\text{Age of universe} \approx 5 \times 10^{17} \, {\rm s}$$

Black hole entropy

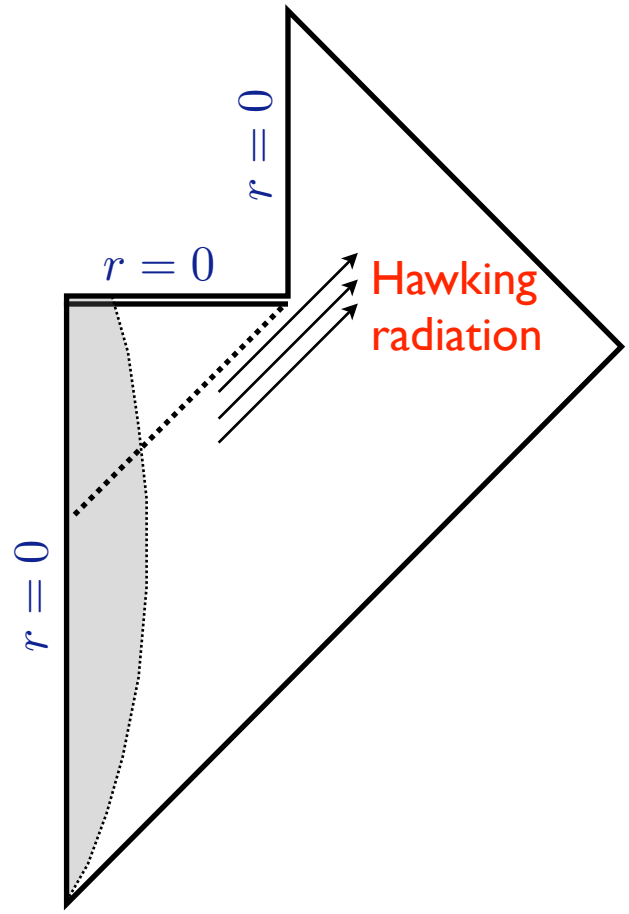
The entropy of a black hole is proportional to the area of the event horizon Bekenstein 1972

Bekenstein-Hawking entropy: $$S_{\rm BH} = \frac{A}{4\ell_{\rm Pl}^2} k_{\rm B} \approx 10^{77} \left(\frac{M}{M_\odot} \right)^2 k_{\rm B}$$

Thermal entropy of the sun: $$S_\odot \approx 10^{57} \, k_{\rm B}$$

Statistical mechanics interpretation of black hole entropy?

Semi-classical black hole



Explicit analytic solutions available for two-dimensional toy models

Callan, Giddings, Harvey, Strominger 1992
Russo, Susskind, Thorlacius 1992

Formulation of the information paradox

Quantum mechanical unitarity

- if a system starts out in a pure initial state $|\psi_{\rm in}\rangle$
- then the final state $|\psi_{\rm out}\rangle$ is also a pure state

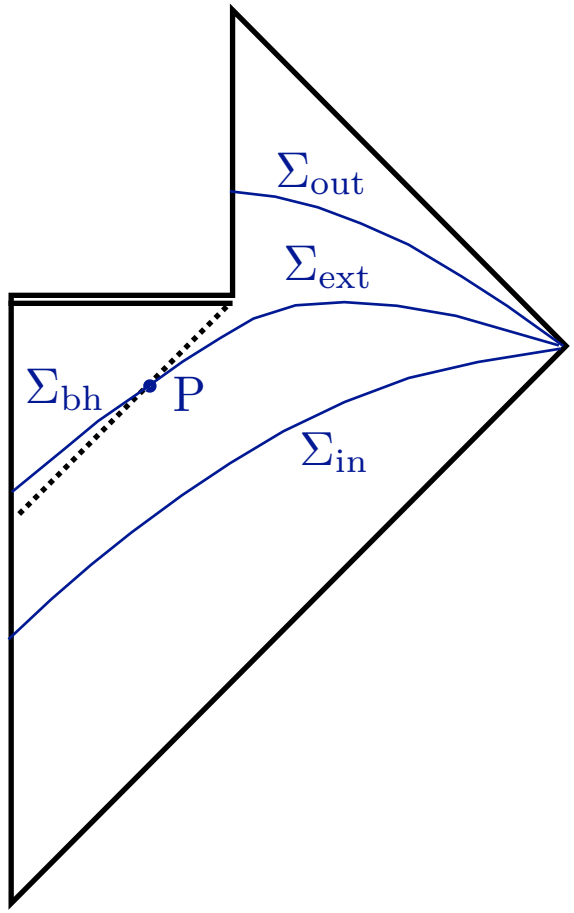
$$|\psi_{\rm out}\rangle = S\,|\psi_{\rm in}\rangle$$

- the S matrix is unitary $\quad S\,S^\dagger = S^\dagger S = \mathbf{1}$
- it follows that $\quad |\psi_{\rm in}\rangle = S^\dagger\,|\psi_{\rm out}\rangle$

The final state carries all information about the initial state

Effective field theory

- assume that local effective field theory can be applied in regions of weak curvature, away from black hole singularity
- the explicit form of the effective field theory is not needed
- construct a convenient set of Cauchy surfaces
  'nice' time slices Wald '93; Lowe, Polchinski, Susskind, LT, Uglum '95
- effective field theory Hamiltonian generates unitary evolution of states



Assume a pure initial state $|\psi(\Sigma_{\rm in})\rangle$

It evolves into another pure state $|\psi(\Sigma_{\rm P})\rangle$ which has support partially inside and partially outside the horizon on $\Sigma_{\rm bh}$ and $\Sigma_{\rm ext}$

Observables with support on $\Sigma_{\rm ext}$ commute with observables on $\Sigma_{\rm bh}$

The state on $\Sigma_{\rm P}$ is therefore an element of a tensor product Hilbert space

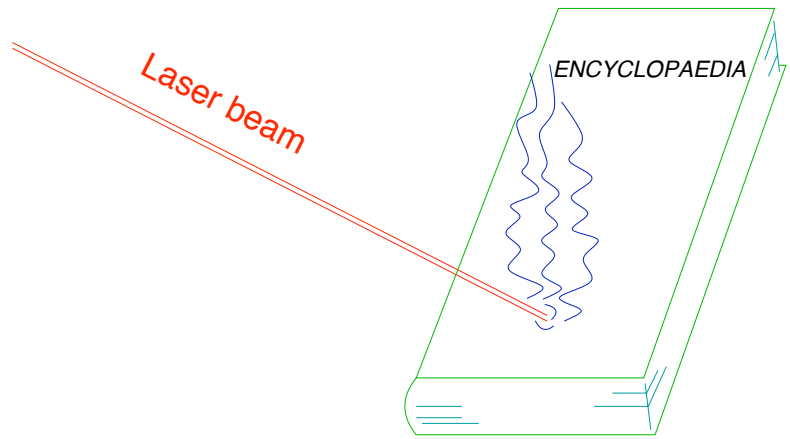
$$|\psi(\Sigma_{\rm P})\rangle \in \mathcal{H}_{\rm bh} \otimes \mathcal{H}_{\rm ext}$$

Taking a trace over $\mathcal{H}_{\rm bh}$ results in a mixed state density matrix on $\mathcal{H}_{\rm ext}$

which will then evolve to another density matrix on $\mathcal{H}_{\rm out}$

The net result is that a pure initial state on $\Sigma_{\rm in}$ has evolved into a mixed state on $\Sigma_{\rm out}$

Information lost to black holes vs. traditional information loss



Emitted radiation appears thermal

For practical purposes information is lost in process

b u t

Final state with outgoing radiation (+ remaining ashes) contains all the information **in principle**

There are subtle correlations between early and late-time radiation

Information carried in the outgoing radiation has been removed from the book

Some suggested resolutions

Non-unitary evolution Hawking '76

- generalized quantum mechanics Hawking '82

Black hole remnants

Planck scale Aharonov, Casher, Nussinov '87 Banks, O'Loughlin '93

macroscopic Giddings '92; Almheiri, Marolf, Polchinski, Sully '12

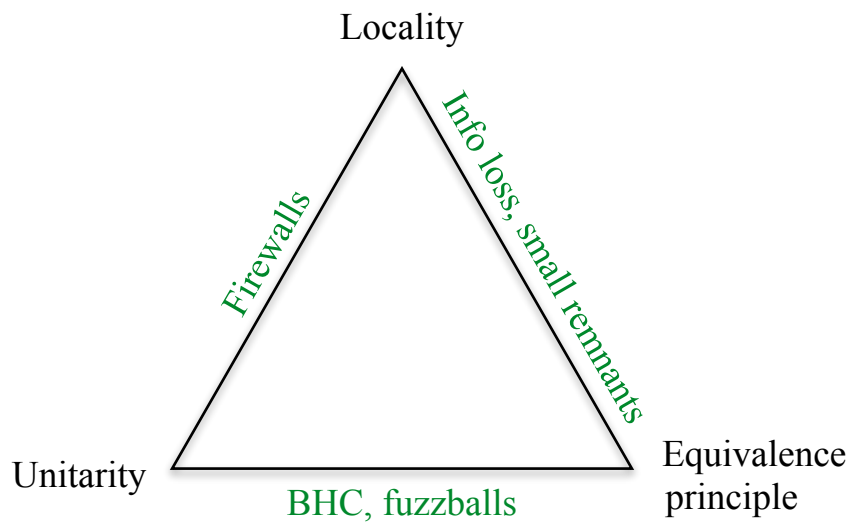
Information returned in Hawking radiation Page '80, 't Hooft '91

- black hole complementarity (BHC) Susskind, LT, Uglum '93
  Kiem, Verlinde, Verlinde '93
- eternal AdS black holes Maldacena '01
- final state projection Horowitz, Maldacena '03

Non-singular quantum geometry

- supergravity fuzzballs Mathur, Saxena, Srivastava '03

Soft hair on black holes Hawking Perry, Strominger '16, '18



Information loss

Purely thermal Hawking radiation implies non-unitary evolution

Hawking ’76

Generalized quantum mechanics Hawking ’82

- replace states by density matrices

- replace S matrix by super-scattering operator $

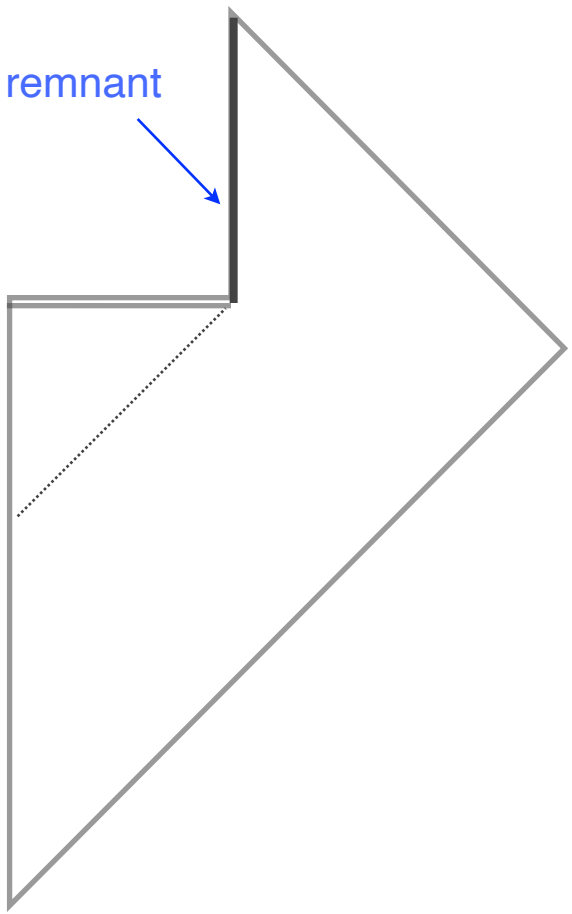
Energy not conserved - vacuum heats up to Planck temperature

Banks, Susskind, Peskin ’84

Ellis, Hagelin, Nanopoulos, Srednicki ’84

Decoherence without dissipation Unruh, Wald ’95; Unruh ’12

Black hole remnants

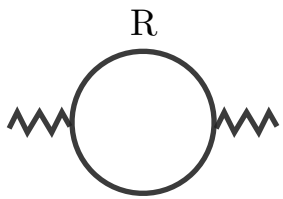


Information about initial state stored in a stable remnant Aharonov, Casher, Nussinov '87

Need a Planck scale remnant for every possible initial black hole

- infinite degeneracy of states

- divergent contribution to quantum loops



$$\text{amplitude} \sim G \sum_R \frac{1}{M_R^2} = \infty$$

Possible loophole: Remnants with large intrinsic geometry Banks, O'Loughlin '92
Hossenfelder, Smolin '09

Unitary black hole evolution

D.N.Page (1980)
G. 't Hooft (1990)
L.Susskind, LT, J.Uglum (1993)
K.Schoutens, E.Verlinde, H.Verlinde (1993)
⋮

Assumptions: Susskind, LT, Uglum (1993)

(1) A black hole is a quantum system with discrete energy levels and finite density of states

(2) The dimension of the subspace of states that describe a black hole of mass M is $\exp S_{\rm BH}(M)$

(3) If the initial state of collapsing matter is a pure quantum state then the system as a whole remains in a pure state at all times

(4) After BH forms, the full system can be divided into subsystems

A - (distant) outgoing Hawking radiation

B - everything else (including BH)

Black hole complementarity

Susskind, LT, Uglum '93

There is no contradiction between outside observers finding information encoded in Hawking radiation and infalling observers entering a black hole unharmed.

- Apparent violation of no-cloning theorem of QM
- Low energy observers in any single reference frame cannot detect duplication of information
- Contradictions only arise when descriptions in very different reference frames are compared
- BHC is consistent with known low-energy physics but implies non-locality and a new degree of relativity in spacetime physics

Tests of black hole complementarity

Membrane paradigm Thorne, Price, MacDonald '82-'86

Replace black hole by a stretched horizon -- a membrane 'near' the event horizon

In astrophysical applications 'near' means close compared to f.ex. distance to companion in a binary system

Quantum mechanical stretched horizon Susskind, LT, Uglum'93

Minimal stretching: $A_{\rm sh} = A_{\rm eh} + 1$

Unspecified microphysics with $\#\ \text{of states} = \exp(A/4)$

Gedanken experiments Susskind, LT '93

Apparent violations of BHC can be traced to assumptions about physics at Planck energy (or higher)

Information paradox involves Planck scale in subtle ways

Input from string theory

Black hole entropy Strominger, Vafa '96

String theory provides a microphysical basis for the entropy of a certain class of (supersymmetric) black holes

$$S_{\rm bh} = \frac{A}{4} = \log\left(\#\text{ of microstates}\right)$$

-- leaves no room for black hole remnants

Gauge theory / gravity correspondence Maldacena 1997

Non-perturbative string theory defined in terms of unitary quantum field theory

-- implementing unitarity on the gravity side remains a challenge

-- large AdS black holes are well-described by finite temperature QFT Witten 1998

-- small AdS black holes are more directly relevant to information problem

Lowe, LT 1999 (+ work in progress)

Unitary black hole evolution

D.N.Page (1980)
G. 't Hooft (1990)
L.Susskind, LT, J.Uglum (1993)
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Average entropy of a subsystem

D.N. Page (1993), S. Sen (1996)

Consider a quantum system with Hilbert space of dimension $m \times n$ in a random pure state.

A subsystem of dimension $m < n$ has average entanglement entropy

$$S_{m,n} = \sum_{k=n+1}^{mn} \frac{1}{k} - \frac{m-1}{2n}$$

For $n \geq m \gg 1$

$$S_{m,n} \approx \log m - \frac{m}{2n} + \mathcal{O}\left(\frac{1}{n^2}\right)$$

Information contained in subsystem:

$$I_{m,n} = \log m - S_{m,n}$$

For $n \geq m \gg 1$ we have $I_{m,n} \leq \frac{1}{2}$

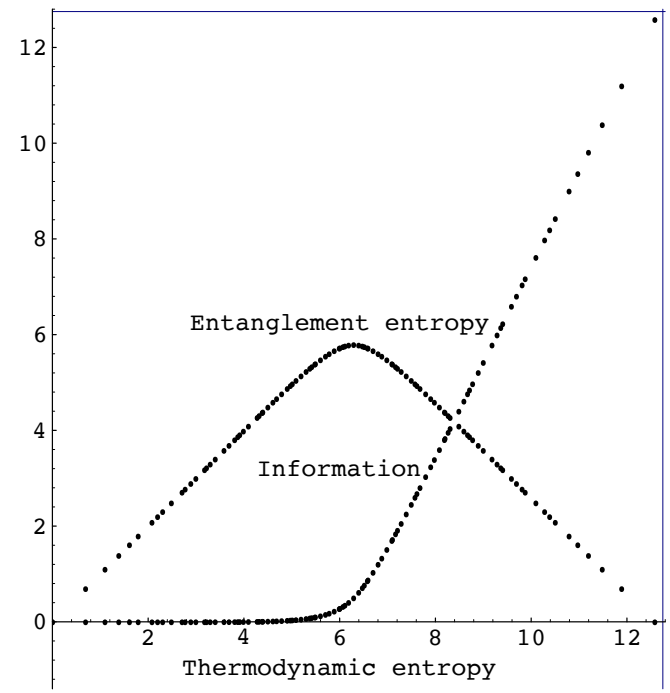
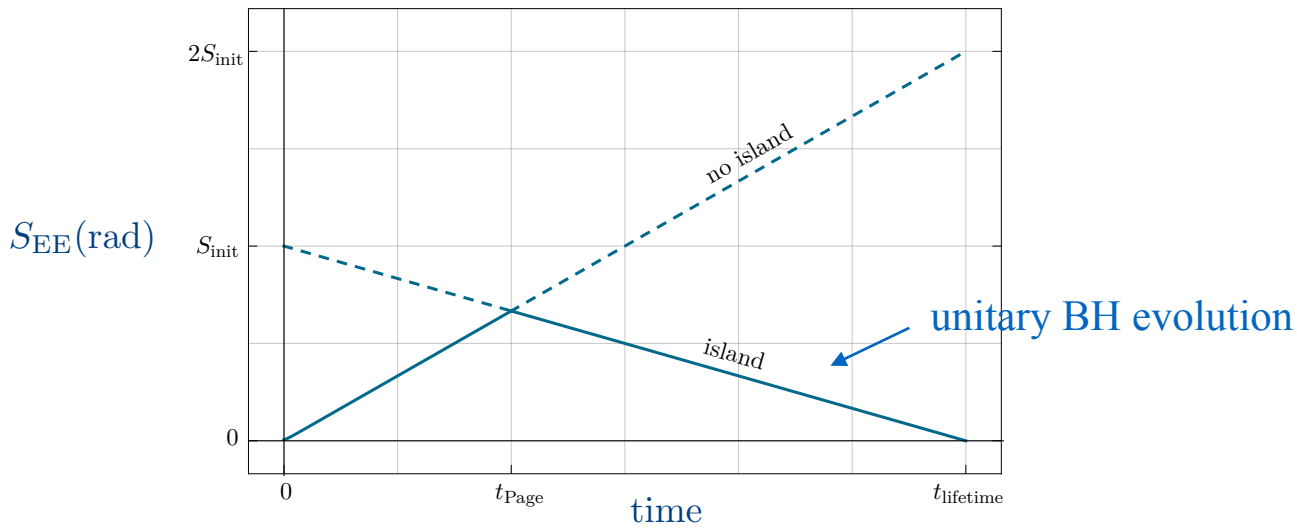


figure from Page (1993)

Page curve for unitary BH evolution

Entanglement between outgoing Hawking radiation and remaining black hole



Can semi-classical results for $S_{\rm EE}({\rm rad})$ be reconciled with unitary BH evolution?

Answer is "yes" if a correction motivated by holographic duality is included

Penington (2019)
Almheiri, Engelhardt, Marolf, Maxfield (2019)

Generalised entropy: $$S_{\rm gen} = \frac{{\rm Area}(I)}{4G_N} + S_{\rm Bulk}[\mathcal{S}_{AI}]$$

Area term dominates after Page time and gives a result consistent with unitarity

Semi-classical theory is surprisingly effective but unitarity remains obscure

Semi-classical Page curves

Black holes in AdS coupled to external CFT

Penington (2019)
Almheiri, Engelhardt, Marolf, Maxfield (2019)
Almheiri, Mahajan, Maldacena (2019)
Almheiri, Mahajan, Santos (2019)
⋮

Extract Hawking radiation via coupling to external CFT Rocha (2008)

Subsystems: rad — external CFT containing Hawking radiation
bh — CFT dual of AdS containing evaporating BH

Full system is in a pure state: $S_{\mathrm{EE}}(\mathrm{rad}) = S_{\mathrm{EE}}(\mathrm{bh})$

Use (quantum corrected) holographic entanglement entropy to evaluate $S_{\mathrm{EE}}(\mathrm{bh})$

Black holes in asymptotically flat spacetime

Gautason, Schneiderbauer, Sybesma, LT (2020)
Anegawa, Iizuka (2020)
Hashimoto, Iizuka, Matsuo (2020)
Hartman, Shagoulian, Strominger (2020)
⋮

Gautason *et al.*:

Adapt semi-classical entropy prescription to two-dimensional dilaton gravity model

Explicit analytic results for semi-classical RST black holes

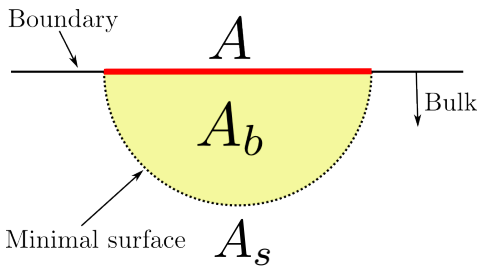
Derivation of generalised entropy formula using replica wormholes

G.Penington, S.Shenker, D.Stanford, Z.Yang (2020)
A.Almheiri, T.Hartman, J. Maldacena, E. Shaghoulian, A. Tajdini (2020)
T.Hartman, E. Shaghoulian, A.Strominger (2020)

Generalized entropy

Ryu, Takayanagi (2006)
Hubeny, Rangamani, Takayanagi (2007)
Faulkner, Lewkowycz, Maldacena (2013)
Engelhardt, Wall (2014)

Quantum corrected holographic entanglement entropy of boundary region A



$$S_{\rm gen}(A) = \min_{A_s}\left\{ {\rm ext}_{A_s}\left[\frac{{\rm Area}(A_s)}{4G_N} + S_{\rm bulk}(A_b) \right] \right\}$$

figure from Faulkner *et al.* (2013)

For a black hole in AdS coupled to external CFT: Penington (2019)

— A is the entire spatial boundary (where dual CFT is defined)

— A_s is a co-dimension two surface *inside* the bulk geometry

For a black hole in asymptotically flat spacetime: Gautason et al. (2019)

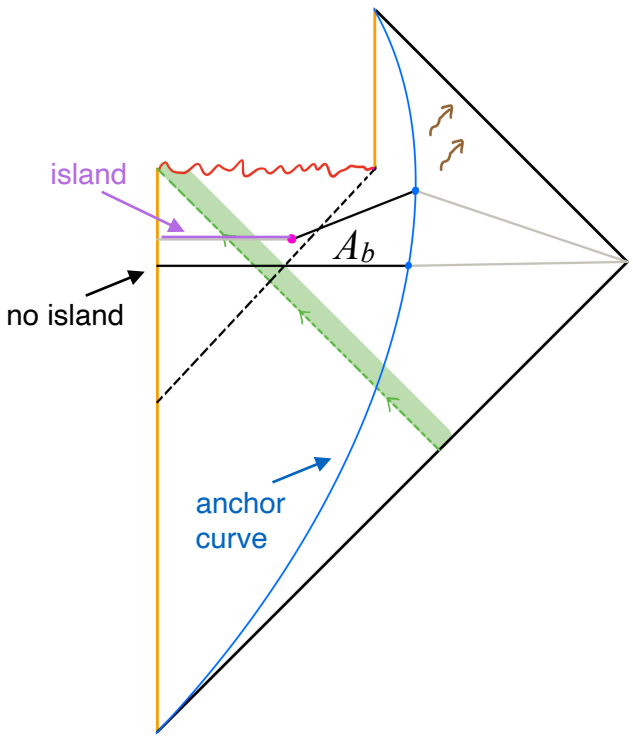
— A is a spatial boundary outside black hole (in asymptotic region)

— A_s is a co-dimension two surface *inside* the bulk geometry

When there is more than one quantum extremal surface A_s we are instructed to choose the one that gives the smallest entropy

Page curve for evaporating RST black holes

Gautason *et al.* (2020)



i) work with solvable 2d semi-classical model

ii) adapt generalised entropy to asymptotically flat background (with linear dilaton)

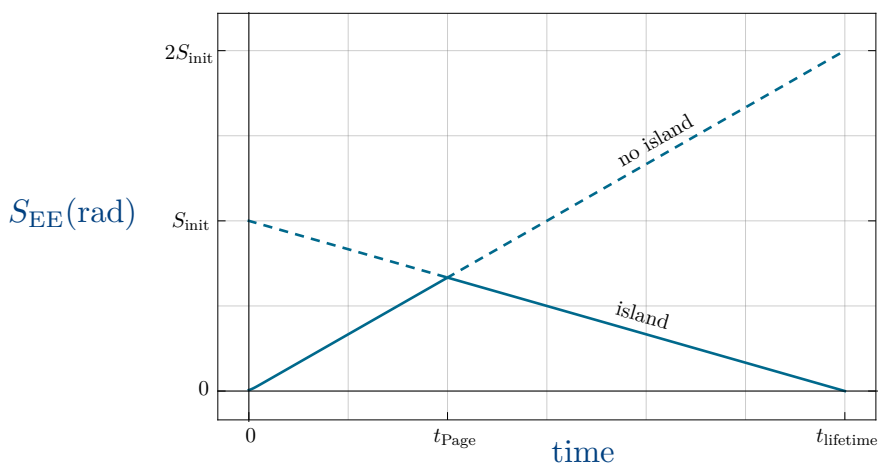
iii) 2d matter described by strongly coupled CFT

→ use AdS_3 holography to calculate S_{Bulk}

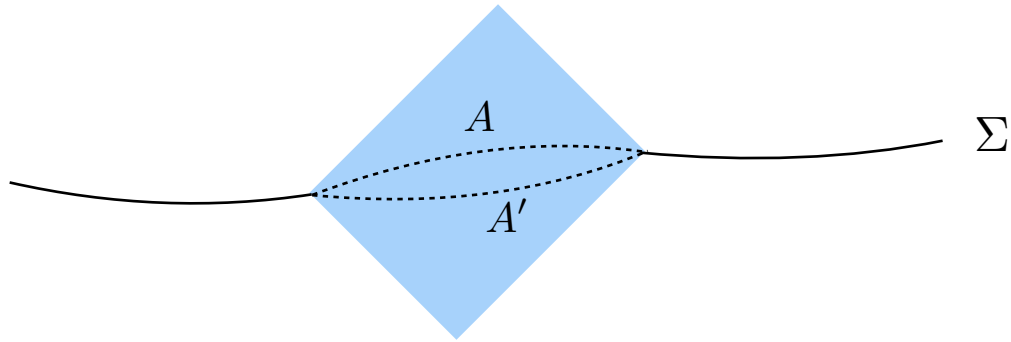
cf. Almheiri, Mahajan, Maldacena, Zhao (2019)

iv) coupling to an external bath is unnecessary - 'inside' and 'outside' separated by anchor curve

v) explicit analytic result for generalised entropy



Entanglement wedges

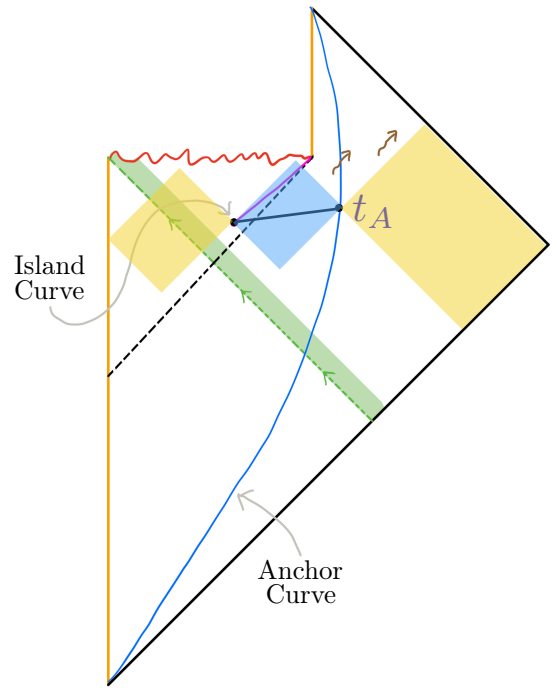


Initial data on A is sufficient to determine bulk fields inside the *causal diamond* of A.

A' has the same causal diamond as A and the same entanglement entropy.

Black hole *entanglement wedge* at time t_A is the causal diamond of a spatial region between point t_A on anchor curve and its QES.

Radiation *entanglement wedge* at time t_A is formed from the causal diamonds of the complement of the spatial region.



Bulk fields in black hole interior

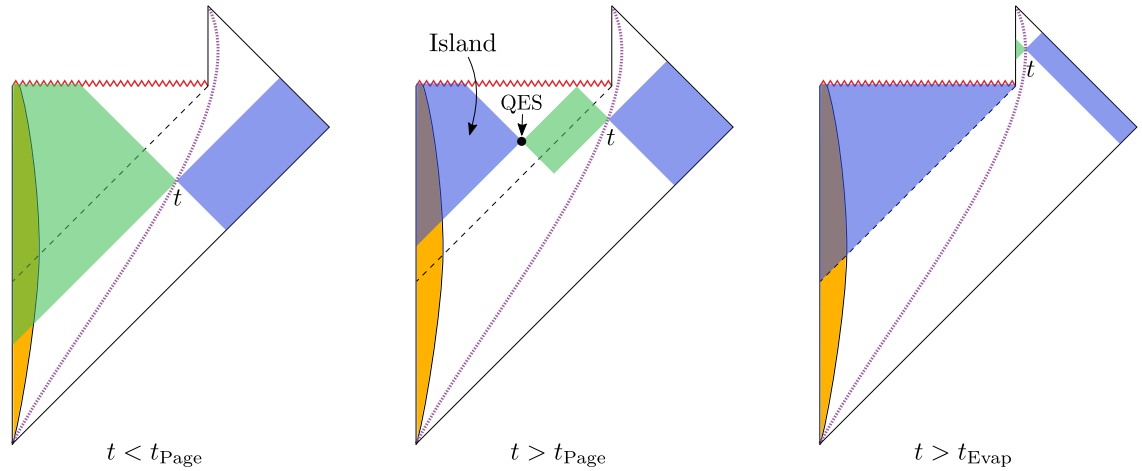


figure from Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini (2020)

Old black hole: ($t_A > t_{Page}$)

Bulk fields on island no longer contribute to generalised entropy of black hole

DOF's on black hole horizon need only describe small part of black hole interior

Do we have enough DOF's to encode observations made by an infalling observer?

— regulate trans-Planckian modes using infalling lattice Corley, Jacobson (1997)

— infalling observers only encounter modes that originate from a scrambling time or less before they enter Lowe, LT (2015)

— toy model for reconstructing bulk fields in the black hole entanglement wedge.

Lowe, LT (2016, 2017)

Holographic quantum complexity

Model BH scrambling dynamics by a quantum circuit with a total number of qubits of order S and a universal set of primitive gates. Hayden, Preskill (2006)

The quantum complexity of a circuit state is the minimum number of primitive gates needed to obtain that state from a given reference state.

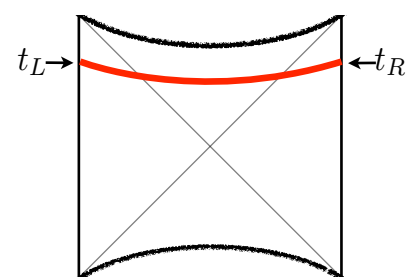
Assuming each qubit gets acted on by at most one primitive gate per cycle we expect $\frac{\Delta \mathcal{C}}{\Delta \tau} \sim S$

or, if each cycle takes of order one unit of Rindler time: $\boxed{\frac{d\mathcal{C}}{dt_S} \sim S\,T}$ $\tau_R = \frac{2\pi}{\beta}\, t_S$

Holographic complexity conjectures:

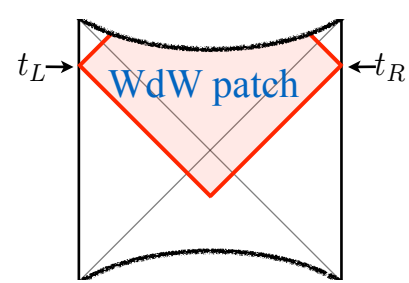
1) Complexity equals volume $\mathcal{C} \sim \frac{V}{G_N R_0}$

Susskind (2014)



2) Complexity equals action $\mathcal{C} = \frac{\mathcal{A}}{\pi}$

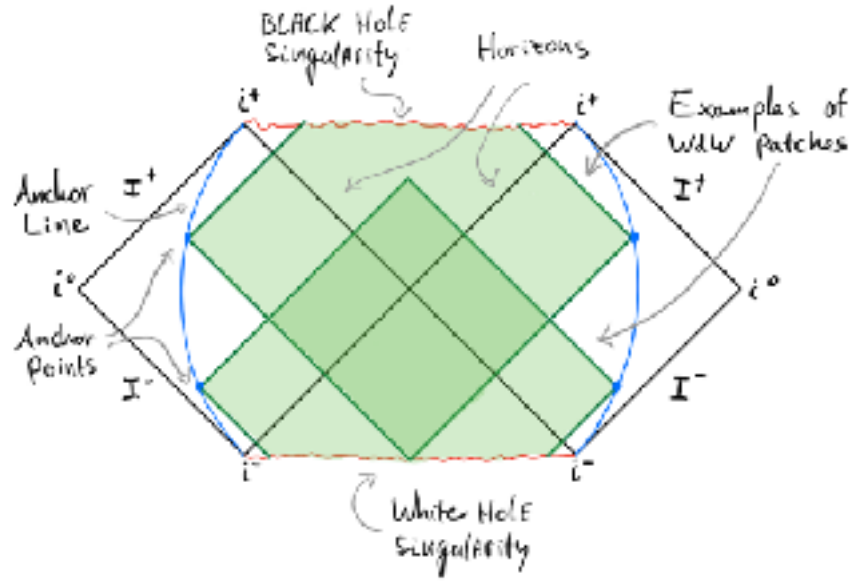
Brown, Roberts, Susskind, Swingle, Zhao (2015)



At late times both 1) and 2) give $\mathcal{C} \sim S\,T\,(t_L + t_R) + \ldots$

C = A : Evaluate action on Wheeler-deWitt patch

Brown, Roberts, Susskind, Swingle, Zhao (2015)



Prescription for gravitational action with null boundaries

Lehner, Myers, Poisson, Sorkin (2016)

Appendix C: Action User's Manual

We include a summary of how to evaluate the gavitational action with all its relevant contributions. We write the gravitational action as

$$
\begin{aligned}
S_{\mathscr{V}} := & \int_{\mathscr{V}} (R - 2\Lambda)\,\sqrt{-g}\,dV \\
& + 2\,\Sigma_{T_i} \int_{\partial\mathscr{V}_{T_i}} K\,d\Sigma + 2\,\Sigma_{S_i} \mathrm{sign}(S_i) \int_{\partial\mathscr{V}_{S_i}} K\,d\Sigma - 2\,\Sigma_{N_i} \mathrm{sign}(N_i) \int_{\partial\mathscr{V}_{N_i}} \kappa\,dS d\lambda \\
& + 2\,\Sigma_{j_i} \mathrm{sign}(j_i) \oint \eta_{j_i}\,dS + 2\,\Sigma_{m_i} \mathrm{sign}(m_i) \oint a_{m_i}\,dS
\end{aligned}
\tag{C1}
$$

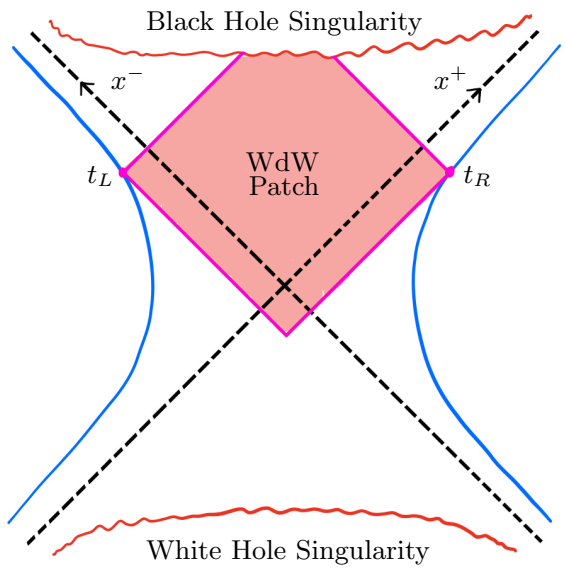
null boundaries
$k^\beta \nabla_\beta k^\alpha = \kappa\, k^\alpha$
$\Rightarrow \kappa = 0$ if λ is affine

Semi-classical black hole complexity

L.Schneiderbauer, W.Sybesma, LT (2020)

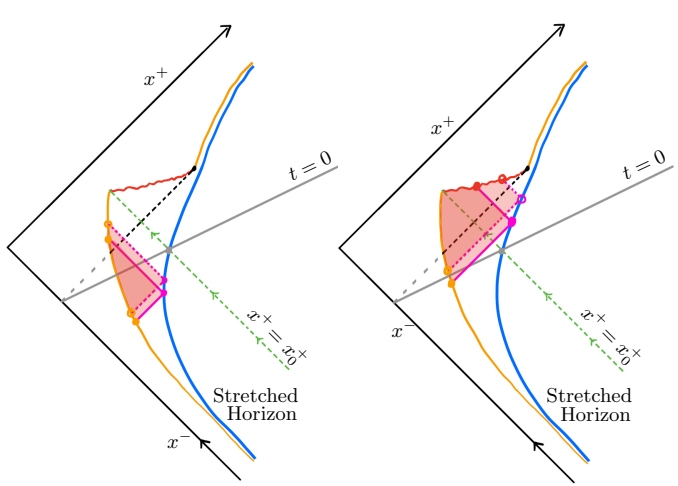
The prescription of Lehner *et al*. can be adapted to semiclassical 2d model

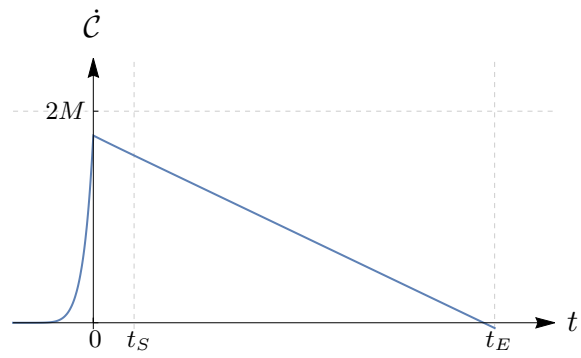
RST static black hole with balanced incoming and outgoing energy flux



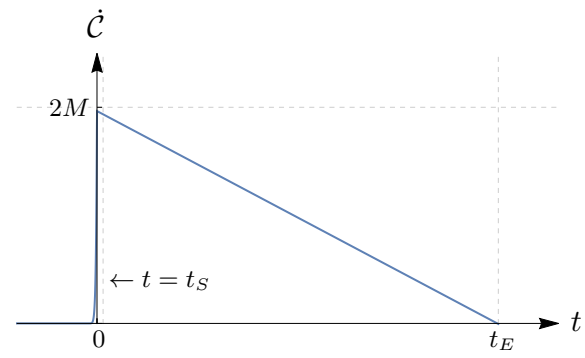
$$\frac{d\mathcal{C}}{dt} = 4M$$

Complexity for evaporating black hole





(a) $M/\kappa = 10$



(b) $M/\kappa = 100$

$$\dot{\mathcal{C}}(t) = 2M\left(1 - \frac{t + 1 + \log\left(\frac{4M}{\kappa} + e^{-t}\right)}{\frac{4M}{\kappa} + e^{-t}}\right)$$
$$= 2M - \frac{\kappa}{2}\left(\log\left(\frac{4M}{\kappa}\right) + 1 + t + \mathcal{O}\left(\frac{\kappa}{M}e^{-t}\right)\right)$$

Semi-classical black hole entropy: $S = 2M - \frac{\kappa}{2}\log\left(\frac{4M}{\kappa}\right) - \frac{\kappa}{2}$

Myers (1994);
Hayward (1994);
Solodukhin (1995)

Rate of change of complexity: $\frac{d\mathcal{C}}{dt_S} \sim ST$

Semi-classical theory is again surprisingly effective

Modeling the black hole interior

#1 : Stretched horizon theory

- description of the interior is non-local and employs finite # of dof's
- area law for black hole entropy implies holographic encoding of BH interior
- emergent semiclassical interior geometry
- so far, no satisfactory theory
- "fast scrambling" horizon dynamics Sekino, Susskind 2008
- toy model: spin system with a nonlocal pairwise interaction D.Lowe, LT '16, '17

#2 : Local effective field theory (extended inside horizon)

- approximate description to match available measurement precision for a typical observer who falls inside black hole
- toy model: bulk Hamiltonian obtained via a mean field construction
- applies on a restricted set of time slices with a radial cutoff in place
- fails for an atypical observer who has measured state of BH

No drama for infalling observer at horizon

Theories #1 and #2 need to have the following properties:

(1) The time required for outside observers to extract quantum information from the black hole (in theory #1) has a lower bound of order the scrambling time.

P. Hayden, J.Preskill '07
D.Lowe, LT '16

(2) From the viewpoint of an infalling observer, who enters the black hole, any quantum information that entered more than a scrambling time earlier has been erased.

Property (2) holds in infalling lattice model D.Lowe, LT '15

Breakdown of bulk description

We want to model a laboratory that falls into a black hole.

Early on the lab is well described by the bulk effective Hamiltonian of theory #2.

The lab has a complementary description in terms of theory #1 and must eventually be absorbed into stretched horizon.

This will appear highly non-local from the viewpoint of theory #2.

In a toy model we find that the decoherence time matches the scrambling time, which is also when lab approaches the singularity.

Result supports the idea that singularity approach is complementary to decoherence of the infalling state.

Summary

The information paradox highlights the incompatibility between general relativity (locality + equivalence principle) and quantum physics (unitarity).

Gauge theory - gravity correspondence implies unitary black hole evolution.

Black hole complementarity provides a "phenomenological" description, which preserves unitarity and the equivalence principle, but requires giving up locality.

Semiclassical gravity based on local effective field theory is nevertheless surprisingly effective

- Page curve reproduced via quantum extremal surface prescription
- rate growth of quantum complexity captured by (semi)classical BH geometry

So far, no satisfactory holographic model for the black hole interior

Requirements include:

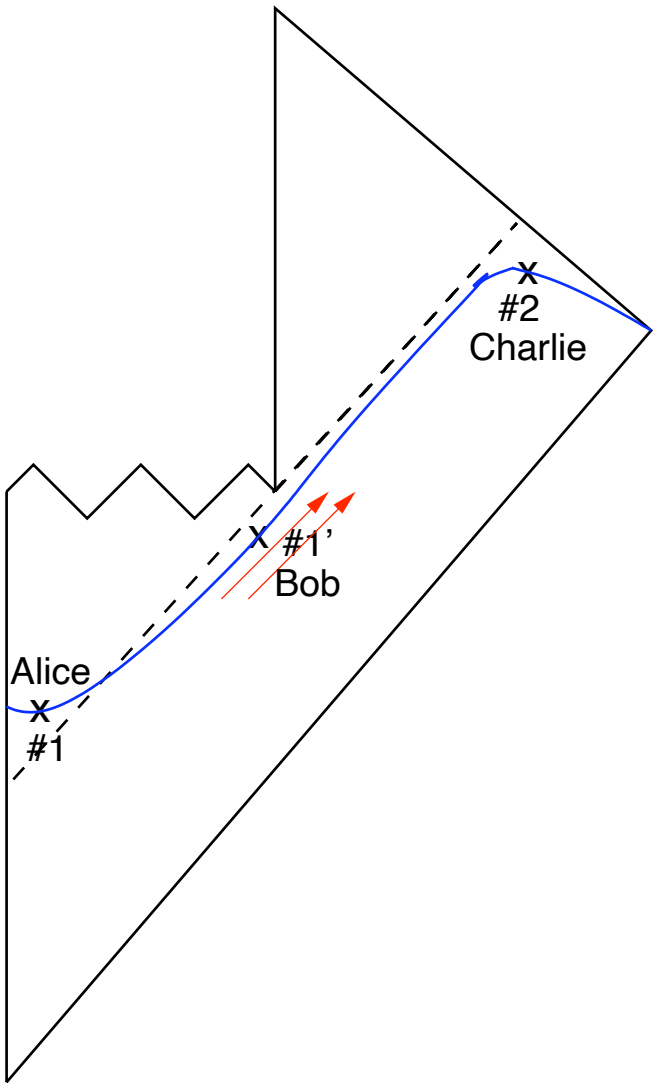
- unitary time evolution of holographic d.o.f.'s
- an approximate description of observers in the black hole interior can be given in terms of an effective field theory, defined on a limited set of time slices, such that no drama is seen until near the singularity

Plenty of work remains…

Formulation of the paradox - v2

Assume EFT is valid on nice slices and carry out a gedanken experiment

Susskind, LT '93; Lowe, LT '06



- prepare singlet pair (#1,#2)

- Charlie takes #2 far away from BH. At an agreed upon time he flips a coin and depending on the outcome he measures #2 along x or z axis

- Alice carries #1 into BH at time of Charlie's measurement and promptly measures #1 along z and broadcasts result

- Bob keeps track of Hawking radiation and measures #1' (quantum clone of #1) along z

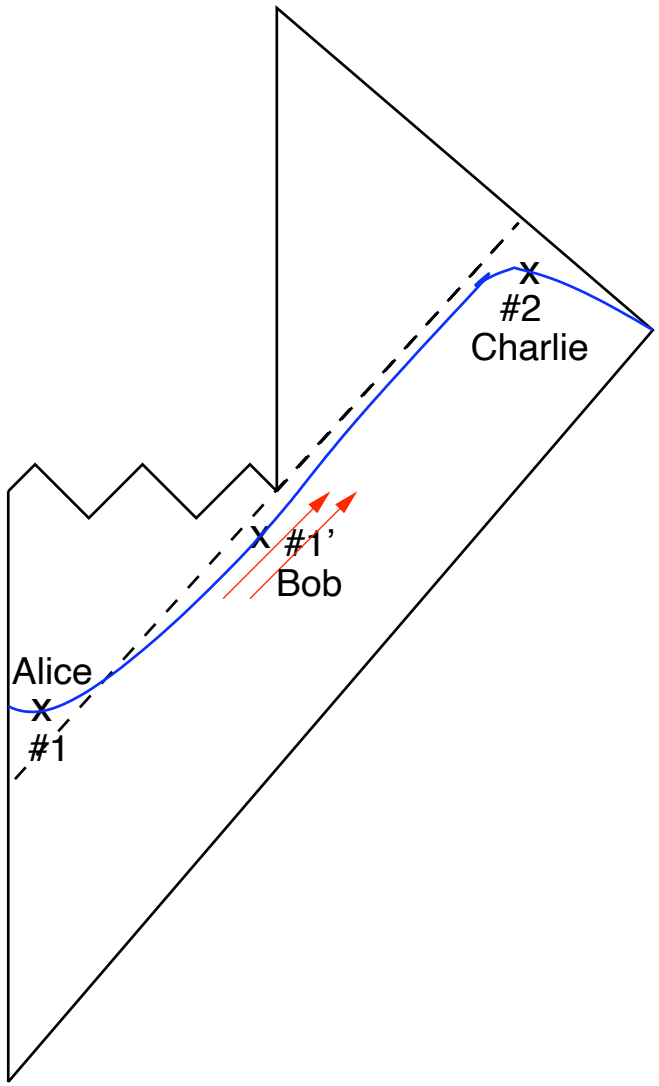
- local EFT : independent measurements by Alice and Bob

- Bob enters black hole and receives message from Alice

- if their measurements disagree Bob discovers that Charlie measured #2 along x axis

$\Rightarrow$ acausal signal from Charlie to Bob

Revisit gedanken experiment



Bob must wait before information can be extracted from Hawking radiation

Young BH: $t \sim r_s \, S_{bh}$ Page 1993

Old BH: $t \sim r_s \, \log r_s$ Hayden & Preskill 2007

Alice has ultra-short time for spin measurement

Young BH: $\Delta t \sim e^{-S_{bh}}$

Old BH: $\Delta t \sim O(1)$

$\Rightarrow$ limited measurement accuracy

Firewall for infalling observers?

Charlie can measure state of Hawking radiation to arbitrary accuracy and projects BH state into eigenstate of his measurement operators

Observation of Hawking radiation burns infalling observer at horizon D.Lowe, LT '06

A. Almheiri, D. Marolf, J. Polchinski, J. Sully '12

S.L. Braunstein, S. Pirandola, K. Zyczkowski '12

RST action on Wheeler-deWitt patch

L.Schneiderbauer, W.Sybesma, LT (2020)

The prescription of Lehner *et al*. can be adapted to semiclassical 2d model

Polyakov-Liouville term is non-local ⟶ introduce auxiliary field Z

$$S_{\rm bulk} = \int_{\mathcal{M}} d^2y\sqrt{-g}\left[R\tilde{\chi} + 4\left((\nabla\phi)^2+1\right)e^{-2\phi} - \frac{\kappa}{4}(\nabla Z)^2 - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right]$$

$$\tilde{\chi} := e^{-2\phi} - \frac{\kappa}{2}(\phi - Z)$$

Boundary action is not uniquely determined

$$\begin{aligned} S_{\rm boundary} = {} & 2\sum_{\mathcal{S}\in\mathscr{S}}\sigma_{\mathcal{S}}\int_{\mathcal{S}}\sqrt{|h|}\tilde{\chi}K d\Sigma \\ & + 2\sum_{\mathcal{N}\in\mathscr{N}}\sigma_{\mathcal{N}}\int_{\mathcal{N}} d\lambda\tilde{\chi}\boldsymbol{\kappa} + 2\sum_{\mathcal{N}\in\mathscr{N}}\sigma_{\mathcal{N}}\int_{\mathcal{N}} d\lambda\,\partial_\lambda\tilde{\chi}\log|\partial_\lambda f| \\ & + 2\sum_{j\in\text{joints}}\sigma_j\tilde{\chi}\big|_j a_j\,. \end{aligned}$$

Require total action to be invariant under the RST symmetry of the bulk theory

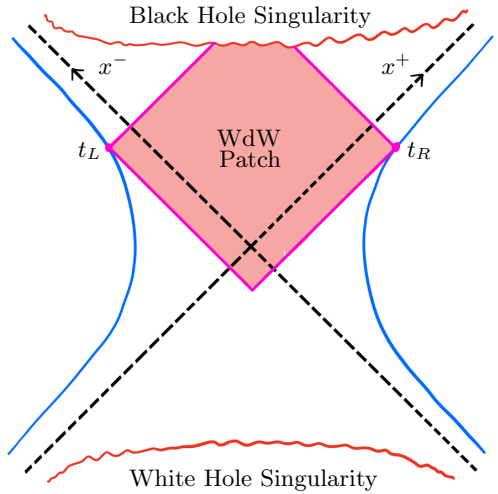
$$\delta_{\rm RST}\,\phi = \delta_{\rm RST}\,\rho = \frac{1}{2}\delta_{\rm RST}\,Z = \epsilon\,\frac{e^{2\phi}}{1-\frac{\kappa}{4}e^{2\phi}}$$

On-shell action takes a simple form in Kruskal coordinates

$$S = \int_{\mathcal{M}} dx^+dx^-\left[2 - \sum_{i=1}^{N}\partial_+ f_i\partial_- f_i + \frac{\kappa}{2}\partial_+\eta\partial_-\eta\right]$$

$$Z = 2\rho + \eta$$
$$\nabla^2\eta = 0$$
$$\delta_{\rm RST}\,\eta = 0$$

Two-sided classical black hole



$$\mathcal{C} = S_{\rm cl} = 2\int_{\rm WdW} dx^+ dx^- =: 2\mathcal{A}$$

$$\boxed{\frac{d\mathcal{C}}{dt} = 4M}$$

One-sided classical black hole

$$\mathcal{C} = S_{\rm cl} = 2\int_{\rm WdW} dx^+ dx^- =: 2\mathcal{A}$$

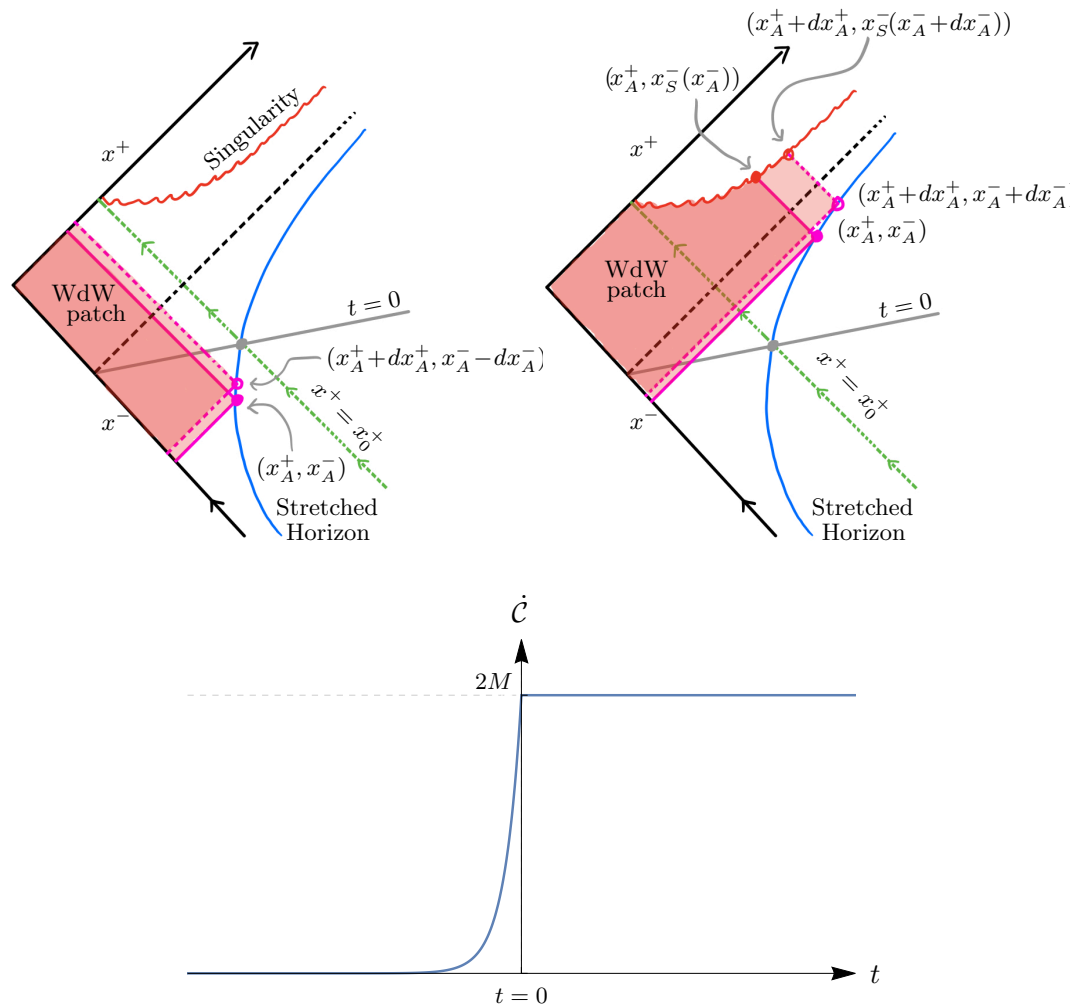


Figure 3. Growth rate $\dot{\mathcal{C}}$ of holographic complexity as a function of tortoise time t, using the CA prescription, for classical gravitational collapse. Following an exponential onset, holographic complexity grows linearly with time.

Summary

- We considered the holographic complexity of black holes in a toy model that allows explicit semi-classical calculations
- C = V maximal volume surfaces
- C = A action on Wheeler-DeWitt patch
- Rate of growth of holographic complexity precisely tracks the shrinking entropy of the stretched horizon
- Area term and subleading log-term in semi-classical entropy are correctly reproduced
- Non-trivial positive test for holographic duality between stretched horizon dof's and black hole interior
- Semi-classical theory is again surprisingly effective!